Comparison of Normal Eigenmodes Calculation Methods Based on Identified Complex Eigenmodes

Qiang Zhang* and Gérard Lallement†
Université de Franche-Comté, Besançon, France

Three calculation methods were recently proposed to transform identified complex eigenmodes of a dissipative structure into normal eigenmodes of an associated conservative structure. These methods were first tested by running a large number of numerical examples and then applied to actual structures. This paper presents the basic ideas of each method. The difficulties encountered during the applications are also analyzed and comments are made on the results obtained. The application of these three methods to a test structure that has two close eigenfrequencies shows that the three results are sufficiently accurate. However, the first two methods use the orthonormality relations and thus require a precise estimation of the identified generalized masses, while the third method, which is iteratice in character, does not use the identified generalized masses and can be recommended for industrial applications.

Nomenclature

$\boldsymbol{B}$	= damping matrix
c	= number of measured points
$\in \boldsymbol{C}^{n,m}$ ($\in \boldsymbol{R}^{n,m}$)	= belonging to complex matrices of order $n \times m$ (belonging to real matrices of order $n \times m$)
$\boldsymbol{I}_n$	= unit matrix of order n
j	$=\sqrt{-1}$
$\boldsymbol{K}$	= stiffness matrix
m	= number of measured modes
$\boldsymbol{M}$	= mass matrix
Re[ ]	= real part of a complex matrix
s_ν	= νth identified complex eigenvalue $s_\nu = -a_\nu\omega_\nu + j\omega_\nu$
$\boldsymbol{y}_\nu$	= νth identified complex eigenvector
$\boldsymbol{z}_\nu$	= νth real eigenvector
λ_ν	= νth real eigenvalue
$[\bar{A}]$	= complex conjugate of matrix A
$[\ ]^T$	= transpose of a matrix
$[\ ]^{-1}$	= inverse of a matrix
$[\ ]^\dagger$	= Moore-Penrose pseudo-inverse of a matrix

Introduction

THIS paper deals only with linear, regular, and dissipative structures, which can be represented by discrete models with symmetrical matrices. The object of the proposed methods is to transform the identified modal parameters, and would later be compared with eigensolutions obtained from a discrete and conservative model of the Finite Element (F.E.) type. Further technical problems such as validation or parametric improvement of the F.E. model can be treated more exactly and efficiently if we make a comparison between normal eigenmodes of the F.E. model and normal eigenmodes of the conservative structure associated to those of the actual dissipative structure.

In particular, for the solution of the difficult inverse problem of parametric improvement of the estimated F.E. model, it would be possible to 1) separate this problem into two independent sub-problems, one concerning the conservative parameters, the other the dissipative parameters, 2) eliminate the difficulties tied to the very bad estimation (or lack of estimation) of the damping matrix in the conservative problem, and 3) work exclusively with real quantities.

Several calculation methods of normal eigenmodes from identified complex ones were proposed.[1-6] They are essentially of two kinds: 1) methods based on the construction of condensed mass and stiffness matrices,[1,2,4] and 2) methods based on the hypothesis of a complex mode representation on an incomplete normal mode basis.[3,5,6]

This paper presents the basic principles of three different methods developed by the Applied Mechanics Laboratory (AML). This theoretical analysis allows us to explain the introduced hypotheses more precisely. It is followed by a discussion of the difficulties encountered during these applications. We assume that the considered mechanical structure is linear and damped, and that damping can be represented by a viscous model. In cases where the structure presents small nonlinearities, the application of Fillod et al.[7] allows us to identify the complex modes of the associated linear structure. This brings us back to the preceding linear structure. In the autonomous case the dynamic behavior of the structure is therefore represented by

$$\boldsymbol{M}\ddot{\boldsymbol{y}}(t) + \boldsymbol{B}\dot{\boldsymbol{y}}(t) + \boldsymbol{K}\boldsymbol{y}(t) = \boldsymbol{0} \tag{1a}$$

where $\boldsymbol{M}$, $\boldsymbol{B}$, and $\boldsymbol{K}$ are symmetrical, positive definite matrices. For $\boldsymbol{y}(t) = \boldsymbol{y}e^{st}$, the associated eigenvalue problem can be written as

$$\left(s_\nu^2\boldsymbol{M} + s_\nu\boldsymbol{B} + \boldsymbol{K}\right)\boldsymbol{y}_\nu = \boldsymbol{0}, \qquad \nu = 1,2,\ldots,m \tag{1b}$$

Eq. (1b) allows us to define the associated conservative structure represented by the eigenvalue problem

$$(\boldsymbol{K} - \lambda_\nu\boldsymbol{M})\boldsymbol{z}_\nu = \boldsymbol{0}, \qquad \nu = 1,2,\ldots,m \tag{2}$$

Solutions from Eqs. (1b) and (2) are grouped into modal matrices $\boldsymbol{Y} = [\cdots \ \boldsymbol{y}_\nu \ \cdots]$ and $\boldsymbol{Z} = [\cdots \ \boldsymbol{z}_\nu \ \cdots]$, and into spectral matrices $\boldsymbol{S} = \mathrm{diag}\{s_\nu = -a_\nu\omega_\nu + j\omega_\nu\}$ and $\Lambda = \mathrm{diag}\{\lambda_\nu\}$. These matrices satisfy the orthonormality relations

$$\boldsymbol{Y}^T\boldsymbol{B}\boldsymbol{Y} + \boldsymbol{S}\boldsymbol{Y}^T\boldsymbol{M}\boldsymbol{Y} + \boldsymbol{Y}^T\boldsymbol{M}\boldsymbol{Y}\boldsymbol{S} = \boldsymbol{D}; \qquad \boldsymbol{D} = \mathrm{diag}\{2j\omega_\nu\} \tag{3}$$

$$\boldsymbol{S}\boldsymbol{Y}^T\boldsymbol{M}\boldsymbol{Y}\boldsymbol{S} - \boldsymbol{Y}^T\boldsymbol{K}\boldsymbol{Y} = \boldsymbol{D}\boldsymbol{S} \tag{4}$$

Received Aug. 26, 1985; revision received March 24, 1986.

*Research Assistant, Applied Mechanics Laboratory, associated with CNRS.
†Professor, Applied Mechanics Laboratory, associated with CNRS.

$$Z^T M Z = I_m \tag{5}$$

$$Z^T K Z = \Lambda \tag{6}$$

The problem we want to solve is as follows: matrices M, B, and K are unknown; and matrices $Y \in C^{c,m}$ $(c \geq m)$ and $S \in C^{m,m}$ are estimated from the application of a modal identification method on a set of measured responses of the structure. From these complex eigensolution matrices, we will determine matrices $Z \in R^{c,m}$ and $\Lambda \in R^{m,m}$, which characterize the eigensolutions of the associated conservative structure.

First Method: Construction of Condensed Mass and Stiffness Matrices

The basic idea of this method has been established by S.R. Ibrahim.[1] He constructs a matrix $M^{-1}K$ from identified complex eigenmodes and then determines the non-normalized eigenmodes by solving the eigenvalues problem

$$\left[M^{-1}K - \lambda_\nu I_c\right] z_\nu = 0$$

For our part, we propose in turn the construction of condensed mass and stiffness matrices from inverse orthogonality relations.[4] In the state space defined by $x(t) = \{y^T(t); \dot{y}^T(t)\}^T$, the two state matrices

$$U \triangleq \left[\begin{array}{c|c} B & M \\ \hline M & 0 \end{array}\right]; \qquad A \triangleq \left[\begin{array}{c|c} -K & 0 \\ \hline 0 & M \end{array}\right]$$

satisfy the following two orthonormality relations

$$X^T U X = \text{block-diag}\,\{\tilde{D}; \bar{\tilde{D}}\}; \qquad X^T A X \triangleq N$$

where

$$N \triangleq \text{block-diag}\,\{\tilde{D}\tilde{S}; \bar{\tilde{D}}\bar{\tilde{S}}\}; \qquad X = \left[\begin{array}{c|c} \tilde{Y} & \bar{\tilde{Y}} \\ \hline \tilde{Y}\tilde{S} & \bar{\tilde{Y}}\bar{\tilde{S}} \end{array}\right]$$

Thus

$$A^{-1} = X N^{-1} X^T$$

Introducing the partitioned matrices A and X in this last equation yields

$$M^{-1} = 2\,\text{Re}[\tilde{Y}\tilde{D}^{-1}\tilde{S}\tilde{Y}^T] \tag{7}$$

$$K^{-1} = -2\,\text{Re}[\tilde{Y}\tilde{D}^{-1}\tilde{S}^{-1}\tilde{Y}^T] \tag{8}$$

$$0 = \text{Re}[\tilde{Y}\tilde{D}\tilde{Y}^T] \tag{9}$$

We then deduce eigenmatrices Λ and Z by solving the eigenvalue problem

$$\left[M^{-1} - \lambda_\nu K^{-1}\right] u_\nu = 0, \qquad \nu = 1,2,\ldots,m \tag{10}$$

and

$$Z = K^{-1}U, \qquad Z \in R^{c,m}$$

where

$$U = [\cdots \quad u_\nu \quad \cdots]$$

Eigenvectors u_ν from Eq. (10) are normalized to satisfy the norm condition from Eq. (5). This condition becomes

$$u_\nu^T K^{-1} u_\nu = \lambda_\nu, \qquad \nu = 1,2,\ldots,m \tag{11}$$

These two methods present the same problem: condensed matrices $M^{-1}K$, M^{-1}, and K^{-1} are of maximal rank m. They are therefore not inversible. To overcome this difficulty, we propose two techniques.

First Technique

We complete modal matrix Y with matrix Y_1, and spectral matrix S with the diagonal matrix S_1 in such a way as to construct the following three matrices

$$\tilde{Y} \triangleq \left[\, Y \,\vdots\, Y_1 \,\right]; \; \tilde{Y} \in C^{c,c}; \; \tilde{S} \triangleq S \oplus S_1; \; \tilde{S} \in C^{c,c};$$

$$\tilde{D} \triangleq D \oplus D_1, \; \tilde{D} \in C^{c,c}$$

Matrices S_1 and Y_1 are chosen arbitrarily under the three following conditions: 1) $S_1 = \text{diag}\{j\omega_{\nu 1}\}$, where angular frequencies $\omega_{\nu 1}$ are outside the band of angular frequencies defined by S. According to Eq. (3), we have $D_1 = \text{diag}\{2j\omega_{\nu 1}\}$; 2) matrix $\tilde{Y}$ is regular; and 3) $\text{Re}[\tilde{Y}\tilde{D}^{-1}\tilde{Y}^T] = 0$ [Eq. (9)].

This condition allows us to satisfy compatibility conditions between direct and inverse orthonormality relations.[4] Matrices $\tilde{Y}$, $\tilde{S}$, and $\tilde{D}$ allow us to build the condensed matrices M^{-1} and K^{-1} defined by Eqs. (7) and (8).

Second Technique

Eq. (9) is solved by successively applying a singular value decomposition on the two matrices K^{-1} and M^{-1}.

The main problems of these two techniques are the following:

1) In general, the condition $\text{Re}[YD^{-1}Y^T] = 0$ is not necessarily satisfied. This means that the structure cannot generally be represented by a condensed model of order m.

2) All the information is contained in matrices Y and S. The introduction of complementary matrices Y_1 and S_1 is a device used to satisfy compatibility, regularity, and sign (positive definite) conditions of matrices M^{-1} and K^{-1}. As neither Y_1 nor S_1 are unique matrices, M^{-1} and K^{-1} will not be either. In fact, we have not been able to analytically prove that the choice of Y_1 and S_1 has no influence on the m desired eigensolutions contained in matrices Z and Λ (even in the case where $\omega_{\nu 1} \gg \omega_\nu$).

Numerical simulation examples show that the accuracy obtained on matrices Z and Λ can be good or bad, depending on the choice of matrices Y_1 and S_1.

3) Norm errors contained in identified eigenvectors y_ν generally have a non-negligible influence over eigenvectors z_ν.

4) The solution of Eq. (10) by techniques based on singular value decomposition or Heisenberg simultaneous triangularization leads to results that strongly depend on eigenform errors and norm errors contained in the identified eigenmatrix Y.

Considering that we do not handle either theoretical analysis or error propagation well enough, we proceed to make use of other principles.

Second Method: Based on Orthogonality Relations

This method is based on the orthogonality relations [Eqs. (3)–(6)] and the following two hypotheses:

Hypothesis 1: $\text{Rank}(Y) = m$

Hypothesis 2: $Y \cong ZT$, $Y \in C^{c,m}$, $Z \in R^{c,m}$, $T \in C^{m,m}$ (12)

This means that the complex eigenvectors can be represented in the subspace of the corresponding normal eigenvectors. Let $Y \triangleq Y' + jY''$; $T = T' + jT''$, where Y', Y'', T', T'' are real matrices.

Hypothesis 2 leads to the following relations:

$$Z = Y'(T')^{-1} \tag{13}$$

$$T'' = T'W \tag{14}$$

$$W \triangleq [Y'^T Y']^{-1} Y'^T Y'' \tag{15}$$

By introducing Eqs. (13) and (14) and hypothesis 2 into Eqs. (3) and (4), and then separating the real and imaginary parts, we obtain the following equations:

$$P = T^T T'; \qquad L = T'^T \Lambda T' \qquad (16), (17)$$

where the symmetrical matrices $P, L \in R^{m,m}$ are known and expressed as functions of matrices Y, S, and W. Matrices T' and Λ are determined by solving the following eigenvalue problem:

$$[L - \lambda_\nu P] v_\nu = 0; \qquad v_\nu^T P v_\nu = 1, \qquad \nu = 1,2,\ldots,m \tag{18}$$

$$\Lambda = \text{diag}\{\lambda_\nu\} \text{ and } V = [\cdots \quad v_\nu \quad \cdots] = [T']^{-1}$$

Then we deduce

$$Z = Y'V \tag{19}$$

This method is based on clear mathematical background. It implies the solution of at least two squares problems [for solving Eq. (13) and constructing the matrix P] and one eigenvalue problem [Eq. (18)], but the dimensions of all the intermediate matrices are small and equal to the number m of identified eigenmodes.

The method's main inconvenience lies in its sensitivity to norm errors contained in y_ν, $\nu = 1,2,\ldots,m$. Theoretical analysis and numerical simulation show that 10% random errors on $\|y_\nu\|$ lead to errors of the same order of magnitude on $\|z_\nu\|$, and 30% random errors on $\|y_\nu\|$ lead to failure of the method. (As an example of consequences of these errors, matrices L or P are no longer positive definite.)

In practice, errors larger than 30% on $\|y_\nu\|$ can be observed, especially in the presence of structural nonlinearities. It is therefore recommended to control or improve the norm precision on identified eigenvectors by means of additional techniques such as the one presented in Zhang et al.[8] before applying such a method of transformation of complex eigenmodes to real normal eigenmodes in which norm equations play an important role.

Third Method: Iterative Method Based on the Homogeneous Equations of Motion

The Method

This method is based on the eigenvalue Eqs. (1b) and (2), obtained from the homogeneous equations of motion, and on previous hypotheses 1 and 2. As the norm relations are not taken into account, the procedure must not be very sensitive to norm errors. In addition, this technique allows us to determine matrices Z, Λ, and the generalized damping matrix, $\tilde{B} \triangleq Z^T B Z$, simultaneously.

Introducing hypothesis 2 in Eq. (1b), we obtain, after premultiplication by Z^T,

$$[s_\nu^2 I_m + s_\nu \tilde{B} + \Lambda] t_\nu = 0 \tag{20}$$

In the $2m$ dimensional state space, Eq. (20) becomes

$$\left[\begin{array}{c|c} -\Lambda & 0 \\ \hline 0 & I_m \end{array} \right] - s_\nu \left[\begin{array}{c|c} \tilde{B} & I_m \\ \hline I_m & 0 \end{array} \right] \left[\begin{array}{c} t_\nu \\ \hline s_\nu t_\nu \end{array} \right] = 0 \tag{21}$$

The basic idea of the iterative procedure follows. The initial estimations $Z^{(0)}$ of Z and $\Lambda^{(0)}$ of Λ are given by

$$Z^{(0)} = Y'; \quad \Lambda^{(0)} = \text{diag}\{\omega_\nu^2\}$$

Then we define matrices $T^{(0)}$ and $\tilde{B}^{(0)}$ as

$$T^{(0)} = Z^{(0)\dagger} Y$$

where $T^{(0)} \triangleq T^{(0)\prime} + jT^{(0)\prime\prime}$ and $Z^{(0)\dagger}$ represent the Moore-Penrose pseudo-inverse of $Z^{(0)}$, and

$$\tilde{B}^{(0)} = A + T^{(0)\prime\prime} \Lambda^{(0)} + \Lambda^{(0)} T^{(0)\prime\prime}$$

where $A \triangleq \text{diag}\{2a_\nu \omega_\nu\}$.

This initial estimation $\tilde{B}^{(0)}$ of the matrix B is obtained by an application of the method of perturbations keeping only the first order terms.

By introducing initial estimations $\Lambda^{(0)}$ and $\tilde{B}^{(0)}$ in Eq. (21), the eigensolutions $S^{(1)} = \text{diag}\{s_\nu^{(1)}\}$ and $T^{(1)} = [\cdots \quad t_\nu^{(1)} \quad \cdots]$ are found. Then $Y^{(1)}$ is calculated by means of relation

$$Y^{(1)} = Z^{(0)} T^{(1)}$$

The differences between matrices $Y^{(1)}$ and Y and $S^{(1)}$ and S are later used to correct matrices $Z^{(0)}$, $\Lambda^{(0)}$, and $T^{(1)}$. The new estimate $\tilde{B}^{(1)}$ is finally calculated by Eq. (20). The details of the method are shown in Ref. 6. This iterative procedure does not stop until the required precision is achieved

$$\|Y\| - \|Y^{(i)}\| = \|Y\| - \|Z^{(i-1)} T^{(i)}\| \le \epsilon \tag{22}$$

where ϵ is a given positive scalar. The flow chart of this method is shown in Fig. 1.

This iterative method presents the following advantages:

1) The generalized damping matrix $\tilde{B}$ and the normal eigenmatrices Z and Λ can be computed simultaneously and therefore coherently.

2) At each step of the iterative process, the identified modal matrix Y can be compared with its reconstitution $Y^{(i)}$, issued from matrices $Z^{(i-1)}$, $T^{(i)}$, $\Lambda^{(i-1)}$, and $\tilde{B}^{(i-1)}$. This comparison can be taken as a precision criterion of the calculated normal eigenmodes.

3) The truncation errors due to hypothesis 2 can be estimated (see the following paragraph).

4) Convergence is rather fast and three or four iterations are sufficient.

We have applied this method to a large number of numerical examples and actual structures. The precision obtained on the normal modes is consistent with that obtained in the identified complex eigenmodes.

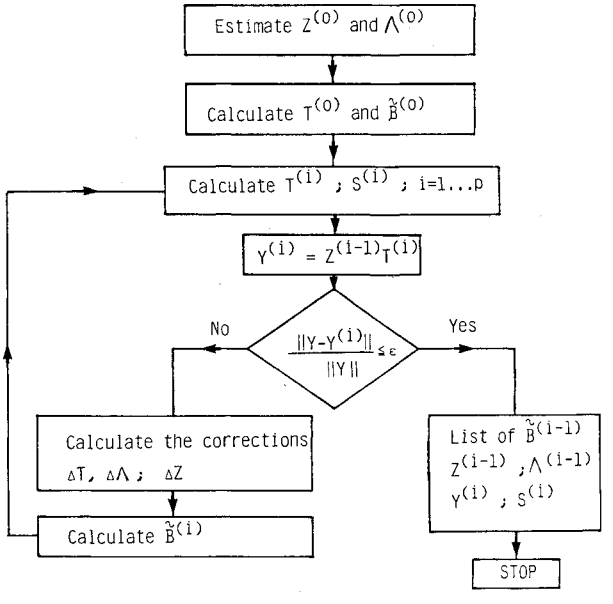

Fig. 1 Flow chart of the iterative process.

This method constitutes an additional processing block that has the role of post-processor for the modal identification software of the linear and nonlinear-mechanical structures developed by AML.

Estimation of Truncation Errors due to Hypothesis 2

In practice, we can only count on a reduced number of identified complex eigenvectors. Normal eigenvectors will therefore be expressed on a fairly incomplete basis, which is why we introduce the truncation errors that are estimated later on.

In Eq. (12), an exact representation of $\boldsymbol{Y}$ would be obtained by projection on a complete basis $\tilde{\boldsymbol{Z}}$

$$\boldsymbol{Y} = \tilde{\boldsymbol{Z}}\tilde{\boldsymbol{T}}, \qquad \boxed{\boldsymbol{Y}} = \boxed{\boldsymbol{Z} \mid \boldsymbol{Z}_1} \boxed{\begin{matrix}\boldsymbol{T} \\ \hline \boldsymbol{T}_1\end{matrix}}, \qquad \boldsymbol{Y} = \boldsymbol{Z}\boldsymbol{T} + \boldsymbol{\epsilon},$$

$$\boldsymbol{\epsilon} \triangleq \boldsymbol{Z}_1 \quad \boldsymbol{T}_1$$

According to the norm defined by Eq. (5), the elements of matrices $\boldsymbol{Z}$ and $\boldsymbol{Z}_1$ are of the same order of magnitude, and errors from truncation (matrix $\boldsymbol{\epsilon}$) can therefore be evaluated from the expression of the general element of matrix $\tilde{\boldsymbol{T}}$. Writing Eq. (20) under the form

$$\left[s_\nu^2 \boldsymbol{I}_c + \tilde{\Lambda}\right] \tilde{\boldsymbol{t}}_\nu = -s_\nu \tilde{\boldsymbol{Z}}^T \boldsymbol{B} \boldsymbol{y}_\nu$$

yields

$$\tilde{t}_{\sigma\nu} = \frac{-s_\nu \boldsymbol{z}_\sigma^T \boldsymbol{B} \boldsymbol{y}_\nu}{s_\nu^2 + \lambda_\sigma} = \frac{-(a_\nu \omega_\nu + j\omega_\nu)\boldsymbol{z}_\sigma^T \boldsymbol{B} \boldsymbol{y}_\nu}{\lambda_\sigma - \omega_\nu^2 + 2ja_\nu\omega_\nu^2 + a_\nu^2\omega_\nu^2}, \quad a_\nu \ll 1 \quad (23)$$

Assuming that $\boldsymbol{z}_\sigma^T \boldsymbol{B} \boldsymbol{y}_\nu = (0.5 \sim 1.5)\tilde{B}_{\sigma\nu}$, we can simplify Eq. (23). We can also accept that in cases where eigenmodes are strongly coupled, the extradiagonal elements $\tilde{B}_{\sigma\nu}$ are of the same magnitude as the diagonal elements $\tilde{B}_{\nu\nu}$, and that in cases where eigenmodes are well spaced, $|\tilde{B}_{\sigma\nu}| < 0.5|\tilde{B}_{\nu\nu}|$. Eq. (23) then becomes

$$t_{\sigma\nu} \cong \frac{-j(0.5 \sim 1.5)\,\omega_\nu \tilde{B}_{\sigma\nu}}{\lambda_\sigma - \omega_\nu^2 + 2ja_\nu\omega_\nu}$$

For $\sigma = \nu$, $B_{\nu\nu} \cong 2a_\nu\omega_\nu$ and $\lambda_\nu \cong \omega_\nu^2 \rightarrow t_{\nu\nu} \cong 1$. The evolution of $|t_{\sigma\nu}|$ has the general form represented in Fig. 2.

Table 1 Measured eigensolutions and calculated normal eigensolutions

	Measured complex modes			
	First mode		Second mode	
Stations	$s_1 = -1.992 + j*209.55$		$s_2 = -2.547 + j*213.40$	
1	$-0.144E+00$	$0.118E+00$	$0.266E+00$	$0.683E-01$
2	$-0.303E+00$	$0.203E+00$	$0.457E+00$	$0.138E+00$
3	$-0.498E+00$	$0.267E+00$	$0.598E+00$	$0.228E+00$
4	$-0.179E+00$	$0.231E+00$	$0.519E+00$	$0.847E-01$
5	$0.210E+00$	$0.207E+00$	$0.464E+00$	$-0.942E-01$
6	$0.618E+00$	$0.192E+00$	$0.433E+00$	$-0.280E+00$
7	$0.106E+01$	$0.192E+00$	$0.416E+00$	$-0.483E+00$

	Method 1	
	First normal mode	Second normal mode
Stations	$\lambda_1^{1/2} = 208.38$	$\lambda_2^{1/2} = 214.22$
1	$-0.146E+00$	$0.227E+00$
2	$-0.301E+00$	$0.384E+00$
3	$-0.484E+00$	$0.496E+00$
4	$-0.195E+00$	$0.449E+00$
5	$0.153E+00$	$0.430E+00$
6	$0.517E+00$	$0.433E+00$
7	$0.905E+00$	$0.447E+00$

	Method 2					
	First normal mode	First recalculated complex mode		Second normal mode	Second recalculated complex mode	
Stations	$\lambda_1^{1/2} = 208.59$	$s_1 = -2.360 + j*209.75$		$\lambda_2^{1/2} = 214.45$	$s_2 = -2.823 + j*213.23$	
1	$-0.141E+00$	$-0.150E+00$	$0.137E+00$	$0.232E+00$	$0.275E+00$	$0.763E-01$
2	$-0.288E+00$	$-0.312E+00$	$0.234E+00$	$0.393E+00$	$0.469E+00$	$0.158E+00$
3	$-0.467E+00$	$-0.513E+00$	$0.306E+00$	$0.511E+00$	$0.615E+00$	$0.260E+00$
4	$-0.183E+00$	$-0.186E+00$	$0.266E+00$	$0.455E+00$	$0.535E+00$	$0.943E-01$
5	$-0.165E+00$	$0.215E+00$	$0.238E+00$	$0.424E+00$	$0.479E+00$	$-0.108E+00$
6	$0.528E+00$	$0.634E+00$	$0.222E+00$	$0.414E+00$	$0.447E+00$	$-0.321E+00$
7	$0.923E+00$	$+109E+01$	$0.214E+00$	$0.419E+00$	$0.430E+00$	$-0.553E+00$

	Method 3					
	First normal mode	First recalculated complex mode		Second normal mode	Second recalculated complex mode	
Stations	$\lambda_1^{1/2} = 208.68$	$s_1 = -1.993 + j*209.63$		$\lambda_2^{1/2} = 214.32$	$s_2 = -2.547 + j*213.32$	
1	$-0.141E+00$	$-0.145E+00$	$0.119E+00$	$0.230E+00$	$0.266E+00$	$0.663E-01$
2	$-0.292E+00$	$-0.305E+00$	$0.203E+00$	$0.391E+00$	$0.455E+00$	$0.140E+00$
3	$-0.473E+00$	$-0.501E+00$	$0.266E+00$	$0.507E+00$	$0.596E+00$	$0.229E+00$
4	$-0.184E+00$	$-0.181E+00$	$0.231E+00$	$0.452E+00$	$0.517E+00$	$0.828E-01$
5	$0.164E+00$	$0.208E+00$	$0.208E+00$	$0.424E+00$	$0.466E+00$	$-0.955E-01$
6	$0.530E+00$	$0.618E+00$	$0.195E+00$	$0.149E+00$	$0.437E+00$	$-0.283E+00$
7	$0.919E+00$	$0.105E+01$	$0.187E+00$	$0.422E+00$	$0.418E+00$	$0.483E+00$

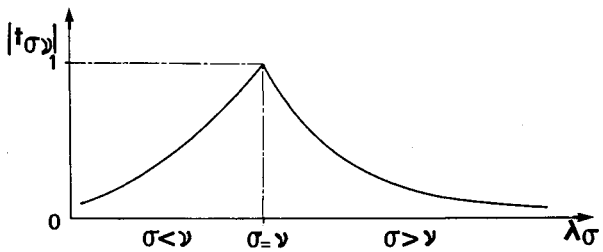

Fig. 2 Evolution of the general element $t_{\sigma\nu}$.

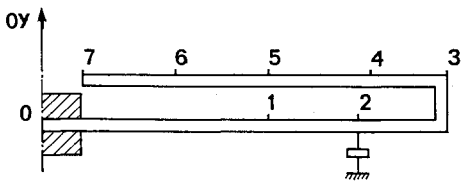

Fig. 3 Folded beam used as test structure.

Table 2 Generalized damping matrix $\tilde{B}$

$\tilde{B}$ by method 2		$\tilde{B}$ by method 3	
4.86	−4.72	4.14	−4.27
−4.72	5.50	−4.27	4.94

In particular, the contribution of the $(m+1)$th mode, generally more important in the truncated part, is

$$t_{m+1,\nu} = \frac{-(0.5 \sim 1.5)\, j\omega_\nu \tilde{B}_{m+1,\nu}}{\lambda_{m+1} - \omega_\nu^2 + 2ja_\nu\omega_\nu}$$

Application

The three proposed methods are applied to the complex eigensolutions of a simple structure (Fig. 3). This test structure is a folded beam excited on its bending eigenmodes along $\vec{y}$. It has the particularity of having pairs of eigenmodes at very closely spaced eigenfrequencies (quasi multiplicity of order two). The eigenshapes are characterized by seven stations numbered from 1 to 7. The damping of the structure can be adjusted and controlled through an analog feedback. The two first complex eigenmodes are identified from a set of forced responses in the frequency domain. The obtained values are reported in Table 1. To these complex eigenmodes, methods 1, 2, and 3 are successively applied in such a way to determine 1) the two eigenfrequencies $\lambda_1^{1/2}$ and $\lambda_2^{1/2}$, and the two corresponding real eigenvectors of the associated conservative structure through method 1, and 2) eigenfrequencies $\lambda_1^{1/2}$ and $\lambda_2^{1/2}$, real eigenvectors z_1 and z_2, and the generalized damping matrix $\tilde{B} \in R^{2,2}$ (Table 2) through methods 2 and 3. The matrix $\tilde{B}$ is determined in two ways: altogether with matrices Z and Λ through method 3; and independently of matrices Z and Λ through a generalization of method 2. This extension is based on a principle similar to that of method 2 and will be described in a further paper. Table 1 also reports the complex eigensolutions recalculated from Z, Λ, and $\tilde{B}$ matrices.

It is noted that these three methods based on three different principles lead, in this example, to very closely spaced eigensolutions. The comparison between identified complex eigensolutions and recalculated complex eigensolutions support the validity of the obtained results.

Conclusion

The three methods that have been proposed make use of fairly different principles. The first method seems to be the simplest but serious gaps subsist in its background. The second one is based on solid foundations but it proves to be too sensitive to norm errors contained in identified eigenvectors. The third method, of an iterative kind, seems to us the most reliable one and therefore has been chosen as a post-processing technique of identified modal data for further applications such as the adjustment of finite element models and structural optimization.

References

[1] Ibrahim, S.R., "Determination of Normal Modes from Measured Complex Modes," *Shock Vib. Bull.*, Vol. 52, No. 5, 1982, pp. 13-17.

[2] Natke H.G. and Rotert, D., "Determination of Normal Modes from Identified Complex Modes," *Zeitschrift für Flugwissenschaft und Weltraumforschung*, 9, 1985, Heft 2.

[3] Niedbal, N., "Advance in Ground Vibration Testing Using a Combination of Phase Resonance and Phase Separation Methods," Proceedings of 2nd ISASD, Aachen, Germany, April 1-3, 1985.

[4] Zhang, Q., Lallement, G., and Fillod, R., "Passage des solutions propres complexes identifiées aux solutions propres de la structure conservative associée," *Mécanique-Matériaux-Electricité*, No. 404, March-April 1984, pp. 18-23.

[5] Zhang, Q. and Lallement, G., "New Method of Determining the Associated Conservative Structure from the Identified Eigensolutions," Proceedings of 3rd IMAC, Orlando, FL, Jan. 1985, pp. 322-328.

[6] Zhang, Q. and Lallement, G., "Simultaneous Determination of Normal Modes and Generalized Damping Matrix from Complex Modes," Proceedings of 2nd ISASD, Aachen, Germany, April 1-3, 1985.

[7] Fillod, R., Lallement, G., and Piranda, J., "Identification of Mechanical Structures by the Smoothing of the Transfer Functions Taking into Account Non-Linearities," Proceedings of 6th IFTOMM, New Delhi, India, Dec. 1983.

[8] Zhang, Q., Lallement, G., and Fillod, R., "Modal Identification of Self-Adjoint and Non Self-Adjoint Structures by Additional Masses Techniques," 10th Conference on Vibration and Noise, ASME, Cincinnati, OH, Sept. 10-13, 1985.